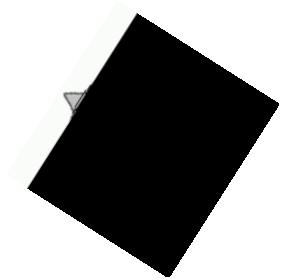


3rd Annual Math Match February 13, 2012

1. Prove that for every $n \in \mathbb{N}$, $n^2 \equiv 0 \pmod{4}$ or $n^2 \equiv 1 \pmod{4}$.
2. Consider a triangular sheet of paper with vertices at the points $(0,0)$, $(1,0)$, and $(0,1)$. By making a single vertical crease in the paper and then folding the left portion of the triangle over the crease so that it overlaps the right portion, the resulting shape is a square. Find the side length of this square.



Solutions or hints:

1. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ so $\frac{1}{4}$

2. $\frac{1}{2} \times x \times x$ -overlap If the fold is at x , then this area is $\frac{1}{2}x^2$.
 maximum occurs at $x = 1$ and equals 2. Hence the minimum area of the desired shape is $\frac{1}{4}$.

3. Consider the function $f(x) = \frac{1}{2}x^2$. Since $f'(x) = x$ then
 there is a $x = 1$ s.t. $f'(x) = 0$. So $f(1) = \frac{1}{2}$ and finally $\frac{1}{4}$.

4. Consider the upper right part of the figure. On the diagram, $\angle A = 60^\circ$ and $\angle B = 60^\circ$.
 We have $\angle C = 60^\circ$. Let $AC = x$ and $BC = y$. Then $AB = x + y$, where $\frac{1}{2}x = y$ from
 properties of equilateral triangle.

Let $AC = x$ and $BC = y$. Then $AB = x + y$ and $\frac{1}{2}x = y$.

Area of $\triangle ABC = \frac{1}{2}xy$ and area of $\triangle ACD = \frac{1}{2}x^2$.