Problems for the 7th Annual Math Match 2020

Solutions:

1. Numbers that are not divisible by 7 may have up to 6 different remainders. By pigeonhole principle, there are 7 numbers with the same remainder from division by 7. So, the sum of these 7 numbers is divisible by 7.

2. Since > 3 is prime than -1 and $+1$ are two consecutive even numbers. So one of them divides by 4 and their product divides by 8. Among the three consecutive numbers -1, , $+1$, one is divisible by 3, but p is not divisible by 3, so one of -1 or $+1$
Must be divisible by 3. Thus, $^2 - 1 = (\ -1)(\ +1)$ is divisible by $8 \cdot 3 = 24$.

3.

Area of $= 25$, as .

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If $=$, then $= 2$. So, $^2 + (2\ )^2 = 25$, which gives $= \sqrt{5}$.
Thus, Area of $2 \cdot = \sqrt{5} \cdot 2\sqrt{5} = 10$.

7. Notice that areas of all the small triangles with base 1 are the same as their height is the same (see the diagram). Denote this area by . Then, the area of the long triangle by the diagonal is 2 . Since the area of the original triangle is 2, then 6 = 2 and = $\frac{1}{3}$